HYPERSEEING

Editors. Ergun Akleman, Nat Friedman.


Page Layout and Cover Design. Ergun Akleman

SUMMER 2014

Cover Images: Photo of a sculpture by Merete Rasmussen and Photos of Upper and Lower Antelope Canyons

Articles

Merete Rasmussen: Hand Built Ceramic Surfaces
Nathaniel Friedman

Birth and Reverberation of an Object
Shane Bearrow and Gabriel Esquivel

Halima Cassell: Carver
Nathaniel Friedman

Challenges with 3D Modeling and Printing Golden Star System
Michael Bunch, Saied Zaririnmehr, Chengde Wu, and Mark Clayton

Larry Frazier: Wood and Stone Bands
Nat Friedman

Movable Sculptures
Michaella Janse van Vuuren

Durer, Drawing and Digital Thinking
Stephen Luecking

Minimal Surface, Sun Dial, and Geodesic Sphere
Nikolaos Georgakas

Books of Interest
Truchet Art
Anna Campbell Bliss

Article Submission

For inclusion in Hyperseeing, authors are invited to email articles for the preceding categories to: hyperseeing@gmail.com

Articles should be a maximum of eight pages.
Introduction

“I want to create a form that you can’t understand until you see the other side. You have to look at it for a while to realize how it is connected.” Merete Rasmussen

Figure 1. (a) Pale Blue Twisted Form, h 30 x w 30 x d 25 cm. (b) Twisted Gray, h 60 x w 60 x d 50 cm.

Merete Rasmussen is a ceramic sculptor who was born in Denmark but grew up in Sweden. She studied ceramics in Sweden, 1997-1999, and then attended Design School Kolding in Denmark, 2000-2005, where she received her Ceramics MA. She has lived in London since 2005. She hand builds elegant clay surfaces that fold back on themselves resulting in equally elegant interior spaces, as in Figures 1 (a) and (b). Their execution takes a high degree of skill.

Artist Statement

I work with abstract sculptural form.

I am interested in the way one defines and comprehends space through physical form. My shapes can represent an idea of a captured movement, as a flowing form stretching or curling around itself, or the idea can derive from repeated natural forms or even complex mathematical constructions. Different form expressions appeal to me and results in my continuous exploration with many different variations: soft but precise curves, sharp edges, concave surfaces shifting to convex; the discovery and
strength of an inner or negative space. I am intrigued by the idea of a continuous surface, for example with one connected edge running through an entire form.

I work with the idea of a composition in three dimensions, seeking balance and harmony. The finished form should have energy, enthusiasm, and a sense of purpose.

The form is emphasized by a monochrome matt surface and I find that strong color builds further importance, strength and energy. My work is hand built in coiling technique in stoneware clay, and mainly unsuitable for outdoor display.

An informative video on her working method is [1].

**Surfaces: Two-sided and One-sided**

The surface in Figure 1(a) has two edges although only one is visible. There is a space through the piece with a second edge. It is two-sided with an inside and an outside. The surface folds in a way that creates an interior space with a complex shape. The surface in Figure 1(b) has one edge and one side. However, this is not clear from the one view alone. This surface also has large interior spaces.

A band in the shape of a trefoil knot is shown in Figure 3 (a). On the left, the outer surface is concave but on the right it is convex. There is only one side and it appears that the transition from concave to convex is hidden behind the wide frontal part of the band.
A striking surface is shown in Figure 3(b). It actually appears that there are two nested surfaces. The surface has one edge but two sides. It is helpful to refer to Figure 2 to understand how the surface is built. Here we see that Merete is building an inner surface that is connected to an outer surface through a central core. That this surface is hand built does not seem possible.

**Enneper Surfaces**

A mathematical surface called an *Enneper surface* is shown in Figure 4(a). A *triple Enneper surface* is shown in (b). A ceramic *triple Enneper surface* by Merete is shown in (c). Blue Double Form can be considered as an ambitious surface inspired by nested *triple Enneper surfaces*. Blue Double Form proves that Merete Rasmussen definitely has superior hands.

Figure 4. (a) **Enneper Surface**  (b) **Triple Enneper Surface**  (c) Ceramic **Triple Enneper Surface**.

Figure 5. (a) **Yellow Hyperbolic Arch**, w 90 x d 50 x h 80 cm.  (b) Side view.
The surface in Figure 5 is a wonderful arch formed by successive hyperbolic shapes. The mathematical term *hyperbolic* refers to the shape of a saddle which has double curvature. If you visualize a saddle, at the center it curves downward where one’s legs are but curves upward at the front and back along the backbone of the horse. A side view is shown in Figure 5. Here it is obvious that this surface has one edge and two sides.

Grey Stretched Form in Figure 6(a) has two sides. Another view is shown in (b). It is like a stretched *Enneper* surface. In (a) one can see that the front center is stretched over to the right creating canyon like interior spaces.

![Grey Stretched Form](image)

*Figure 6. (a) Grey Stretched Form, w 60 x h 55 x d 45 cm. (b) Side view.*

**Spiral Forms with One Side and One Edge**

Grey Twisted Form in Figures 7 (a) and (b) has one side and one edge. This is not possible to see since the center structure is not clear. There is five-fold rotational symmetry. However, the structure is quite impressive and the five bowl-like components that spiral around required a lot of skill to get just right.

Grey Green Twisted in Figure 8 is another spiral form with one side and one edge. This form has three-fold rotational symmetry. The outer shell like form results in three bowl shape spaces. The center structure is intricate and one can see how the outer surface of the top bowl blends into the outer surface of the lower left bowl, as well as into the inner surface of the lower right bowl. It follows by three-fold symmetry that the outer surfaces of the bowls are connected and also connected to the inner surfaces of the bowls. That is, the outer surface is connected to the inner surface so the surface is one-sided. The property of being one-sided is of mathematical interest. The primary interest is in the elegant spiral form of the surface, as well as the interior spaces.

A selection of four additional surfaces with one edge and one side are shown in Figures 9(a)-(d).
Figure 7 (a) Grey Twisted Form, h 55 x w 55 x d 45 cm.

Figure 7 (b) Grey Twisted Form, Side view.

Figure 8. Grey Green Twisted Form, h 45 x w 45 x d 45 cm.

Figure 9 (a) Blue Twisted Form, h 60 x w 60 x d 45 cm.

Figure 9 (b) Orange Form, h 35 x w 35 x d 35 cm.

Figure 9 (c) Blue Twisted Loop, h 60 x w 60 x d 50

Figure 9 (d) Blue Twisted Loop (Wall)
Bronze Sculpture (untitled)

Merete was commissioned to create a sculpture for the Beechfields View Housing Development in Torbay, Devon, GB. Three views of her bronze sculpture are shown in Figure 10. Her comments are as follows: “I have often lived by the sea, so the sea has been important in my life. The sculpture is a continuous surface with an unbroken line running through the whole form like the waves. My aim was to capture the form, texture and color of the coast near here. ” MR

The sculpture was first sculpted in wax and then the sculpture was cast in bronze by the lost wax process. This sculpture proves that her work is powerful at the large scale.

Figure 10. h 130 x w 130 x d 90 cm., Torbay, Devon, GB.

Conclusion
The elegant ceramic surface sculptures of Merete Rasmussen are unique. Just a small number have been discussed above and additional images and information is available on her website www.mereterasmussen.com and on Google. Her working method is described well in [1]. We would like to express our gratitude to Merete Rasmussen for allowing her images to be inserted in this article.

Reference
Birth and Reverberation of an Object
Shane Bearrow¹, Gabriel Esquivel²
Texas A&M University Department of Architecture

Abstract
“Birth and Reverberation of an Object” is, in part, an analysis of Graham Harman’s object-oriented ontology, through which a natural object is stripped of its ontology through a series of craft iterations. The basic idea of this project was to conduct a series of drawing exercises going from analog to digital in order to produce a unique shape. This process was inspired by Robin Evans’ essay “Translations from Drawing to Buildings.” All steps in the process were unique, though clearly traceable and geared toward the autonomy of an architectural object. Similar to De l’Orme’s Diane de Poiters interior, diagrammatic parallel projections guided the object through instances of dimensional, textural, and shape shifting before it reached its final destination. Through a drawing-governed evolution, an object was born governed by methodological iterations, thus the use of the word “reverberations.” This object exhibits transplanted characteristics of its source while appearing strange and difficult to read, ultimately enhancing its appeal.

Introduction
We are in a moment where architecture is redefining its position, moving from a subject-centered and systematic discourse to an object-oriented situation. Objects need not be natural, simple, or indestructible. Instead, objects will be defined only by their autonomous reality. They must be autonomous in two separate directions: emerging as something over and above their pieces, while also partly withholding themselves from relations with other entities (1). Object-oriented ontology (OOO) is a metaphysical movement that rejects the privileging of human existence over that of nonhuman objects (2). Specifically,
object-oriented ontology opposes the anthropocentrism of Immanuel Kant's Copernican Revolution, whereby objects are said to conform to the mind of the subject and, in turn, become products of human cognition (3).

Harman’s object-oriented ontology opens up a unique possibility for rethinking the peculiar problems associated with the problem of nature. A return to the object would have to be understood as a turning away from a mythological or sentimental understanding of nature toward the particularities and the essential strangeness of the objects themselves. In this particular project, the use of a seashell, an object of nature, was a deliberate selection. By submitting this “natural object” through a series of drawing translations, a new object related to its autonomous drawing process rather than nature was created. This object doesn’t operate in normative representation.

Assume for a moment that the architectural object is unified as an object, and remember that an architect is also an object in this ontology, not an enlightened mind outside the world of objects giving form to formless matter (4). A return to the architectural object as a disciplinary priority cannot be a nostalgic return to pre-modern academic preoccupations with character, propriety, and the idealities of a compositional balance. Nor is this return to the object a simple return to figuration and detached massing. “Object” here should not be understood in a literal sense.

Successful object making cannot be completely encapsulated by a methodology that might repeat the success. There are diverse methodologies to investigate. This object operates outside of formal indexical operations. As a non-theoretical interaction between the maker as an object and the various objects of the making process, “craft” is the ambiguous word that has, in the past, identified the unique expertise of the maker in the relationship to the material. This where the relationship between Evans’ position in regards to drawing in terms of inventing complex drawings is what we have referred to as the architect’s craft and the object-oriented ontology that allows for the theoretical revisions of the future of an architectural object.

**Translations of Drawing to Architecture**

“Drawing in architecture is not done after nature, but prior to construction; it is not so much produced by reflection on the reality outside of drawing.”

Robin Evans

In the essay “Translations from Drawing to Building,” Robin Evans argues that the hegemony of drawing over the architectural object has never been challenged. The discussion goes further into introducing the idea that the architecture drawing does not operate in classic representation but precedes the architectural object, creating a complex relationship between objects—drawing and object. This project relied on the ability to flatten and abstract, to distance itself from the object, and to produce friction between objects and their mutual representations.

Evans discusses a precedent that influenced this process. The characteristics and relationship between the dome and paving pattern of the Royal Chapel at Anet by de l’Orme upon close analysis are deceptively difficult to describe using structural, geometric, or stylistic terminology. The “expansion of lozenges, rib thicknesses and angles of intersection” were determined protectively, through the rigorous use of stereotomic diagrams generating the complex, hippopede governed patterns through an extrusion of simple, familiar line work (5). The intricate dome articulations were then projected into the pavement and enlarged, and then excess overlap was removed, allowing the pattern, and thus projections, to move full circle (two dimensional [2-D] to three dimensional [3-D] and back to two dimensional; see Figs. 1 and 2). Through this commission, a new mode of architectural drawing was established in which a diagrammatic
matrix was installed, strict though malleable parameters were constructed, and unpredictable yet controlled and intentional results were obtained.

![Fig 1. Dome. Anet—Le château de Diane de Poitiers. By De L’Orme, Copyright © Argazkilari 64, 2010, all rights reserved, reproduction interdite, même partielle. On explore 20 janvier n° 285.](image1)


According to Evans, “Despite the possible astronomical roots or symbolic backing of the parallel projections, the architectural meaning and likeness are preserved through the process making the transportation from idea to construction successful and compelling” (6).

This project used a similar process of dimensional translation. While the Royal Chapel at Anet started two dimensionally, the drawings were ultimately derived from three-dimensional mediums that were, in consequence, flattened in Pepakura, producing the official point of diagrammatic departure. Also similarly, once the pattern was transferred through the first parallel projection channel creating a three-
dimensional object, a rectilinear, seemingly flattened diagram followed. Whereas l’Orme’s project went full circle once, the present model did so twice, cycling through various iterations of dimensional status and quality of line work, removing from the object its familiarity without sacrificing its integrity. While the dome and paving of Anet look deceptively similar (despite the removal of excess intersections), the iterations of Birth and Reverberation look deceptively different (7). The digital and craft-oriented shifts are motivated completely by the shell as an object, despite the suggestion of the seemingly alien end product.

Methodology

Cut Lines.
The project began with the observation of seashells in an attempt to understand their characteristics in terms of form, acoustic properties, and surface texture. The next step involved selecting which seashells would be modeled in Maya, initially replicating dimensions, form, and surface nuances (see Fig 3).

![Fig 3. Original Seashells and Maya Models.](image)

Then the shells were flattened in Pepakura, which translates 3-D data into a 2-D printable format, stripping their ontology through the removal of qualities attributed to seashells. The methods of capturing the model are often unique to the subject and the tools available. In this case, the textures and the wireframe model were, as mentioned earlier, created in Maya and arranged in Rhino before being exported to Pepakura. Before exporting the file from Rhino, all higher-order geometry was converted into polygon meshes (see Fig 4). The 3-D model needed to have correct (outward-facing) normals and correct (counter-clockwise, right-hand-rule-out) polygon vertex order, and adjacent polygons had to meet in a water-tight fashion with no cracks or edge vertices that belong to only one polygon (see Fig 5).

After this, the model was refined to give the proper layout and construction tabs that would affect the overall appearance and difficulty in constructing the model. The properties that were modified were the following:

a. Three dimensional became two dimensional (several planes vs. one plane).
b. Course/rough became smooth.
c. Curves became rectilinear.

Fig 4. Process from selected seashells, the first Maya models, and the unfolded flattened model.

Three-dimensional moments were then injected back into the Pepakura planes, giving the model the appearance of a “cubist” fabric. Once the file was ready, the 2-D version was printed out on 110 lb cardstock and cut out using a laser cutter. This mass reinterpretation suggests that rather than using digital technology to reverse engineer the construction of the complex form, the digital tools should become a mechanism to better understand material and fabrication potential. It is an argument for developing and refining precise parametric systems of material properties, tooling/fabrication behaviors, and construction
contingencies and using them to forward engineer use and architectural form. It is in essence an argument for architects to begin developing systems of constructional knowledge, by and through which we can design.

In general, the areas of the original model with the highest **absolute Gaussian curvature** generated the most deviation in the Pepakura design and required the most additional cut lines. In general, a flat plane, or any surface extruded from a 2-D curve, will have zero Gaussian curvature; however, the shell models that were used had some absolute Gaussian curvature. The flattened model was imported into Maya, and the process of cut lines and line work began (see Fig 6).

![Fig 6. Relationship between cut lines and surface line work in Maya.](image)

Fig 7. Three flattened objects with their corresponding Maya Models.
The three shells were merged through line work and the connection or rhythmical movement of one nuance into another. The fabric was then fractured, and the pieces were reassembled, giving mass and spatial variation to the object. Using the rigid line work of the Pepakura model, fragments were stitched (see Fig 7).

The most important guideline for the next exploration back to Maya was the use of the cut lines from the Pepakura model. These lines were transferred to the Maya polygonal surface as line work that would be interpreted as edge loops to give specific definition and articulation to the digital 3-D model.

**Line Work and Edge Loops**

Once in Maya, an important part in developing proper edge loops involved understanding poles. The E(5) Pole and the N(3) Pole were the two most important poles in developing both proper edge loops and a clean topology on the model. The E(5) Pole was derived from an extruded face. When this face was extruded, four 4-sided polygons were formed in addition to the original face. These pole theories allowed the reinterpretation of the line work into edges for definition and creasing to be more precise. The criteria for connecting the sequence included the next edge in the sequence being the \((i + 2)\)nd edge of the shared vertex, determined in order from the current edge \((i)\) (8).

**Fig 8.** Complete non-linear process from the seashells, flattened objects, Pepakura model, and final composite of new ontology Maya model.
The dimension in terms of how high the edge loops should be pulled was taken from the Pepakura model. Several commands were used to work with the edge loops; the Select Edge Loop Tool and Select Edge Ring Tool, as well as the Offset Edge Loop Tool and the Insert Edge Loop Tool, use a variety of criteria for the selection of the various edge types, depending on the specific model and modeling style. Users can crease or harden the edges on their polygon meshes (see Fig 7). Hardening or creasing the edges determines how the mesh transitions between faces, enhancing the realism of the model (9). The final goal was that the object be strategically tightened through subdivision modeling. The final result was an object that was a product of the “craft” of drawing and with a new ontology different from the seashells (see Fig 8).

After the new object in Maya was developed from the composite of the three original seashells, several sections were cut to study the object even further (see Fig 9).
The final step was the pattern application that came from a 2-D study of the shells and their flattened patterns. The textures and nuances of various distorted seashells were flattened uniformly, generating fragments of line work that were bridged strategically to remove any doubt of unity (see Fig 10). This pattern was applied to the object using the stencil function within ZBrush. Group loops were generated based on this pattern, allowing the crisp application of a shell-reminiscent color scheme. Noise was applied to selected brushstrokes as well as the texture of the object, reinstating the porosity discarded during the primary Pepakura flattening phase. Initially, the qualities essential to the object were removed, only to be returned in different proportions and states throughout its evolution (see Fig 11).

![Fig. 10. Preliminary pattern to be imported to ZBrush.](image)

![Fig. 11. Final Object with ZBrush Pattern.](image)
Conclusion and Future Work

This project involved generating an object that departed from nature by changing its ontology at the end of the process. Use of the drawing craft presents alternatives for architecture to apply normative modes of production in a different way, through the combination of analog and digital presentations in 2-D and 3-D. The most immediate future development that we will undertake is to fabricate our object using the CNC milling machine to create a base form made out of foam. From this positive mold, we will create the final object using composite materials, epoxy resin, and c-glass; the ZBrush pattern will be unfolded from the digital model using Pepakura, and it will be printed and incorporated as a layer within the composite surface.

After the prototype is built, we will do a structural analysis using ABACUS, a software used by aerospace engineering, and determine the performative needs and properties of the object’s surface. Additionally, we are currently researching materiality concerns and construction techniques on an architectural scale. Once the properties of each material are deduced, our focus will shift to solving various technical challenges of fabricating parts of the object full scale.

References

6. Ibid. p. 181
7. Ibid. p. 175
Introduction

When I first saw all the work credited to Halima Cassell, I thought there were at least three Halima Cassells as in Figures 1 (a), (b), and (c). It turns out there is only one very prolific Halima Cassell. She was born in Pakistan in 1975 and moved with her family to Manchester, England when she was one year old, where she was brought up. She obtained an undergraduate degree in art in 1997 and an MA in 2002. She now lives in Blackburn, Lancashire. Her work has been aptly described as “a fusion of her Asian roots with a fascination for African pattern work and a passion for architectural geometry.”

![Halima Cassell: Carver](image)

Figure 1. One Halima Cassell carving (a) Clay, (b) Marble (c) Wood.

A new technique for ceramic sculpture

Sculpture is generally modeled in clay, carved in wood, stone, or plaster, welded or cast in metal. Halima Cassell introduced an original technique of first modelling an initial clay form, such as a thick bowl, and then carving into the surface when it is between leather hard and completely dry. An initial bowl shape already provides a circular hollowed out form on which to carve a design that is initially a two-dimensional drawing on the concave surface. Moreover, she also draws a two-dimensional design on the outer convex surface and then carves into the outer surface. The result is a completely three-dimensional carving all around the bowl shape on both the inner and outer surfaces. There is no front or back and the inside and outside are equally interesting. In particular, there are strong light and shadow effects resulting from the carving process which consists of separate strokes of a knife. The basic circular form thus results in a sculpture that invites one to move around with no one preferred viewpoint. Rather there is a continuously changing 360 degree range of images that have no beginning and no end. It is totally three-dimensional. Examples are shown in Figure 2 (a) and (b).
The pieces displayed below are an installation of sculptures. The individual clay bodies are acquired from different parts of the globe and differ in color and texture, as shown in Figure 3.

This original technique of preparatory modeling followed by the main work of carving has resulted in a very widely recognized body of work.
Halima Cassell has really opened the gates to a whole new world of stoneware sculpture. For example, in Figures 4 (a) and (b) she is shown working on vertical sculptures. In Figure 5, she is working on a large relief sculpture. Additional reliefs are shown in Figure 6.
Stone Sculpture

We note that a very talented stone carver might attempt to carve a Cassell bowl sculpture directly in stone. This may or may not be possible since carving both sides without fracturing is very difficult. In any case, Cassell first carves the sculpture in nearly dried clay with a knife and then fires the clay to obtain a stoneware sculpture. Thus she carves the clay and then turns the clay into stone.

Having said that, eventually Halima Cassell was inspired to carve directly in stone. In this case she carves the outer surface of the stone block using a pneumatic chisel, as in Figure 7(a). This is an intermediate stage in the carving of the sculpture Calliope shown in the three views in Figures 7(b-d). In Calliope, we see that when carving in stone, Cassell can carve quite deeply into the stone so that she is carving deep spaces and the corresponding light shadow effects are very strong. The sculpture is truly a composition of form, space, light, and shadow. In Figure 7(b) we can even see a little opening at the top where Cassell has carved through the stone. As in most of the bowl forms, the sculpture is completely three-dimensional and changes dramatically as one’s viewpoint changes. Calliope is discussed in detail in an enlightening interview with Halima Cassell at [1]. A general interview is at [2].
An impressive marble carving in process is shown in Figure 8(a). Here we see the surface pattern that guides the carving. Another beautiful marble carving is shown in Figure 8(b).

Wood Sculpture.

As shown in Figure 1 (c), Cassell also carves wood sculptures using traditional hand tools. This sculpture *Navel* is shown in Figure 9(a). However, for the sculpture *Unfurling* shown in Figure 9(b), a chain saw was used. Here Cassell was not concerned with surface patterns but rather with a composition of strong vertical forms.
Iron and Resin Sculpture

A tall iron and resin sculpture *Fan Construction* is shown outdoors in Figure 10(a) and an indoor detailed view is shown in Figure 10(b). *Fan Construction* was originally carved in clay and a mold was made from the clay sculpture. The location is Cartwright Hall, Bradford, UK.

Bronze Sculptures

Halima Cassell has also had a selection of sculptures cast in bronze. The sculptures are first carved in an initial material such as clay or plaster and then a mold is made in order to make a bronze casting. Two bronze sculptures are shown in Figure 11.

The large impressive bronze sculpture *Makonde* in Figure 12(b) was originally carved in plaster, as shown in Figure 12(a). The mold for the bronze sculpture was then made in silicon and a hard case shell. *Makonde* is activated by a certain figurative lean. As in *Calliope* in Figure 7, *Makonde* consists of a composition of deeply cut spaces resulting in a variety of form space light shadow relationships.
that change as one moves around the sculpture. Cassell’s general abstract sculptural style emerges strongly in *Calliope* and *Makonde*.

![Figure 11](image1.png)

**Figure 11.** (a) *Crystalline*, 2008, 12 in. diameter (b) *Staccato*, 2012, 12 in. diameter.

**Cast Lead Crystal Sculpture**

Cast lead crystal sculptures can be made in the same way that bronze sculptures are made from a mold of an original. A beautiful crystal bowl is shown in Figure 14. The mirror image is a perfect enhancement.

**Conclusion**

We have now presented a small selection of works by Halima Cassell in clay, stone, wood, and bronze. This should convince you that she is “A vital force to be reckoned with”. Her website http://www.halimacassell.com is a very extensive source of information on her work. There are two books corresponding to exhibits of her work [3] and [4] with informative texts. In conclusion, we wish to express our gratitude to Halima Cassell for allowing the insertion of the images in this article.
References


![Figure 13. Amoeba Pool, 2012, 12 in. square, Cast lead crystal glass.](image)
Golden Star System is an experimental structural system which uses biomimicry to bring together the geometric configuration of the Koch Star, the Golden Mean and the rotation of DNA around its centerline. Creating physical models of Golden Star System (figure 1) appeared to be a challenge. This paper introduces a parametric application to generate Golden Star System around a desired curve in Rhino-Grasshopper and presents a detailed process to make the model ready for 3d printing.

This project focused on an experimental system called the Golden Star System. It involves the nexus of the Golden Mean, the Koch Star (Snowflake) and the rotation of DNA around the center of its axis. The hypothesis associated with this design is that the nexus of these components will improve structural load carrying capacity in dynamic loading situations. Tests are currently being planned. The Golden Mean ratio has been studied for millennia and it has been shown to reoccur repeatedly in nature. Nature has used it to develop efficient systems to support structural loads with minimum materials. In addition the Koch Star has been similarly studied for dozens of years. Both in their charterer are similar at the macro
and micro level. We see the golden mean repeated in sunflower seed distribution, nautilus shell construction, the ratio of energy needed to survive as it relates to the mass of animal [2], and nearly everywhere we look in our global groups of living things. We also see it in the rate of rotation of DNA and the distance between its strands.

In order to accomplish these exciting new design ideas, we need to develop an application to generate prototypes of their geometries. We intended to develop and application which could generate the Golden Star System around any given spatial curve. Since the curve might have any irregular geometry we needed a parameterized application to generate it. However, the existing generation capabilities of this model had specific requirements which could not be addressed by the Grasshopper native batteries alone. In addition, Grasshopper batteries produced unnecessary outputs. The application that was used to develop the solution was the Visual Basic (VB) battery within Grasshopper 3D that uses an open NURBS (Non-Uniform Rational B-Splines) library in Rhino. By the selection of specific desired parameters, a real time visualization of the desired model was generated. We selected scripting because it allowed more control over the form generation process.

**Form Generation Algorithm**

In order to generate desired geometry we first had to develop an appropriate algorithm to produce the different components of the form. The process begins with a curve on the central axis of the Golden Star System. The length of the axial curve was set as the golden mean ratio and the number of desired modules were then defined (figure 2). This design consideration required that we use only complete modules. Our application calculates the allowed number of modules around a prescribed axial curve. If one selects a number of modules that is greater than those which meet these parameters, an error message will be displayed as it exceed maximum length. However, if we desire to construct a model which uses less than the maximum number of modules, this application will allow the user to place it at the apex of the arc or at each end of the axial curve (figure 2).

![Figure 2: Placement of the Golden Star Along a Given Axial Curve](image)

The method used to draw the helix is based on finding a sequence of points on the helix and connecting them together using a spline. In order to develop the desired smoothness, the degree of our NURBS curve instance should be set at two. With a degree higher than one, the helical NURBS curve will not contain all of the points between the starting and ending points. Some level of error is unavoidable. The level of accuracy can be increased or decreased as required by each project. We accomplish this by either increasing the number of points or changing the degree of the spline. The application allows you to set your desired level of accuracy. Each point on the helix has a corresponding point on the axial curve and vice versa. For each of the points on the axial curve, a plain was drawn perpendicular to the axial curve (figure 3). The directions of the generated plains are random. As such we need to align them (figure 3).
After aligning the plains, it is necessary to determine how much to move and rotate the center of the local coordinate system in each plain to find the corresponding helix point. The rotation angle is proportional to the length of the curve at that specific point. We can choose the rotation angle that can be multiplied by a global rotation factor in order to change the number of rotations of the helix (figure 4). Adding an initial global rotation angle to each point will cause all of the points to revolve around the axial curve of the helix. When we increase the rotation we do not change the distance between the points that generate the helix. This is to maintain the same level of accuracy for all helix components (figure 4). Additional parameter inputs are required to determine the direction of the rotation.

In figure 5 you can see how the components of the Golden Star System are added together. We constructed a single helix by adding the central axis (red), the helix component (green), the equilateral triangle (green), the star components (blue), and the triangular ties (yellow) in sequence. In figure 6 this profile, highlighted in red, is adapted to a curve to create an arch-like golden star system. This figure shows how Golden Star System combines similar profiles to create a complex geometry. Apart from the
helix and the axial curve, all of the other components in figure 5 were drawn using polylines whose vertices were generated based on the geometry of the helix and the axial curve. Each of the components in figure 5 was also generated independently by finding their control points. As shown in figure 5, each point on a given vertex, such as point A, has corresponding points projected on the helix (point B), and the axial curve (point C). Accordingly, unique \( x \), \( y \) and \( k \) values can be found based on the distance between point C and D, point C and B, and point B and A.

![Figure 7: The Function Locates Points Based on Their Length & Offset Values in the Profile](image1)

![Figure 8: Changing the helix Geometry for Additional Components](image2)

We can define a programming function to find these polyline vertices based on their \( x \), \( y \) and \( k \) values on the flat profile (figure 7). This function will draw a plane (green) on a point of the axial curve at which the length to the starting point of the golden star system equals to \( x \). This plane will intersect with the helix at B. When the axial curve has a complex geometry the plain shown in figure 7 intersects with the helix at more than one point. In those cases our point-finding function finds point B by filtering out the rest of the points whose distance to C is greater or less than \( y \). After finding point B and C, we can easily find A by translating C along the direction of CB and to the length of \( k \). Using this function all one needs to do is to read the flat profile, find the corresponding \( x \), \( y \), and \( k \) values of the vertices, find corresponding points on the space, and finally draw a polyline to connect these points to each other. Figure 8 demonstrates how a profile can be generated using this technique. It also shows how changes in the helix will affect the shape of the other components.

In the conceptual design, the components of the Golden Star profiles are intended to be pipes. Therefore, we considered all of the lines and curves as the central axis of pipes. We also wanted to 3d-print the geometry. Preparing the geometry for 3d-printing asked for the generation of a valid Standard Tessellation Language (STL) file out of the complex combination of pipes. In so doing, we faced a lot of challenges that will be explained in the following section along with our solutions to overcome them.

**Repairing STL file for 3d printing**

STL (Standard Tessellation Language / STereoLithography) is a file format commonly used in 3d printing, an additive digital fabrication. STL files only contain shape information (no color, material, etc.). An STL file is simply composed of the data for a sequence of triangular faces: the normal (right-hand rule) of each face and xyz coordinates of the three points of the face (figure 9). Each triangle, as shown in figure 10, must follow Vertex-to-vertex rule [1].

There are two kinds of STL files: ASCII, shown in figure 11, and binary, shown in figure 12 (Burns, 1993). The file size of ASCII STL file is approximately five times of binary STL file because of their different data structures. Although both ASCII and binary types equally qualified for 3d printing, not all
STL files are printable. Three basic rules make an STL file printable. STL files must be watertight (figure 13, 1), all faces facing outward (figure 13, 2), and non-self-intersecting (figure 13, 3). Self-intersecting objects are not allowed, but one STL file can contain multiple good (valid) meshes that intersect each other.

1&2. Naked edge and non-manifold edge. Each edge in a good mesh must connect two faces, no more or less than two. An edge connecting more than two faces is called a “non-manifold edge”. On the other hand, an edge connecting less than two faces is called a “naked edge”. Self-intersecting objects cause non-manifold edges at the intersection and leaky (not water-tight) objects cause naked edges at the opening. In figure 14, green edges (3 and 4) are good edges, orange edges (1 and 2) are non-manifold edges, and magenta edges at the peripheral are naked edges. Edge 3 connects two faces, face C and D, but edge 1 connects three faces, Face A, B and C.

3. Disjoint pieces. A “disjoint pieces” object has multiple pieces in one mesh object (figure 15). In other words, each piece in a “disjoint piece” object is not able to be selected independently. The important difference between a “disjoint pieces” object and a self-intersecting object is that a self-intersecting
object has at least one non-manifold edge but in a “disjoint pieces” object, two or more pieces do not share any edges (no non-manifold edges).

4. **Inconsistent normals** (Faces that would improve their results if their directions were flipped). Because all normals of the faces should be outward, any face with inward normal will cause “Inconsistent normals” problem. (figure 13, 2)

5. **Degenerate face.** Degenerate face happens when three vertices of the face are collinear even though they are distinct points [3]. In figure 16, vertex A, B, and C forms a degenerate face (“triangle” ABC) because these three vertices are collinear. At a glance, face 1 (triangle ACD) looks failed to comply vertex-to-vertex rule (figure 10). However, face 1 complies with the rule because of the existence of degenerate face ABC.

6. **Zero length edge.** Degenerate faces sometime have zero length edges. In figure 16, if vertex B and C are at the same position, edge BC is a zero length edge.

7. **Duplicate faces.** As the name implies, duplicate faces are two or more identical faces at the same position. In figure 17, exactly aligned two identical cubes will cause duplicate faces shown in orange. At the same time, multiple non-manifold edges will occur.

![Figure 14: Types of edges](image1)

![Figure 15: A “disjoint pieces” object (left) and a self-intersecting object (right, two non-manifold edges in orange)](image2)

![Figure 16: Degenerate face ABC](image3)

![Figure 17: Duplicate faces](image4)

After looking at these problems, we figured out a set of solutions to repair the bad meshes in our Golden Star System components. We used Rhino with “MeshRepair” plug-in as the repairing tool. Repairing a bad mesh usually takes more than one step, which makes it confusing. Both the numbers of steps and the order are crucial. Repairing a bad mesh can be compared to starting a car; starting the engine, stepping on the clutch, changing gears, releasing the hand break, and stepping on the accelerator while releasing clutch. Missing a step or switching the order of the steps will cause the car to stop. The 3D printing of the Golden Star System required repairing the mesh. When repairing a self-intersecting mesh that is made of two pipes with non-manifold edges (figure 18, 1) for the Golden Star System, the plug-in transformed the mesh into disjoint pieces with naked edges (figure 18, 2). Continued repair of the disjoint pieces is required so that the meshes were converted into two independent pipes with naked edges (figure 18, 3). When we filled the holes, the pipes became two good meshes (figure 18, 4). However, as we were
saving the file in STL file, Rhino automatically joined two good meshes into one bad mesh with non-manifold edges. This was exactly the same file as we started with. By using a “Meshbooleanunion” command to unify the two meshes into one before saving the file we were able to prevent it from automatically converting into a bad mesh again (figure 18, 5).

Figure 18: A bad mesh with non-manifold edges (1), disjoint pieces with naked edges (2), two independent meshes with naked edges (3), two good meshes (4), and one good mesh after “Meshbooleanunion” (5)

Followings are detailed solutions to each specific error:

1. Naked edges. Use of the “Fillmeshhole” command to fill holes one at a time, or use of the “Fillmeshholes” to fill multiple holes is possible. The “Fillmeshhole” or “Fillmeshholes” may distort original shape. When this occurred we found that it was better to use “Patchsingleface” to build each triangular face (figure 19). If the hole is too small to see its location on the screen, we used the “showedges” command which allowed us to render the naked edges in a distinct color

Figure 19: Original mesh with non-manifold edges (1), after deleting non-manifold edges (2), “Fillmeshholes” command distorts the shape (3), using “Patchsingleface” to repair the naked edges (4)

2. Non-manifold edge. With the use of the “ExtractNonManifoldMeshEdges” command or Mesh Repair plug-in we can extract non-manifold edges. Then by deleting the extracted faces with non-manifold edges the original mesh will have naked edges. Accomplishing this task is a correct step toward developing a good mesh.

3. Disjoint pieces. Use “SplitDisjointMesh” or Mesh Repair plug-in to split it into multiple meshes.

4. Inconsistent normals. Use “UnifyMeshNormals” or Mesh Repair plug-in to repair flipped faces.

5. Degenerate face. By the use of the “CullDegenerateMeshFaces” command or Mesh Repair plug-in to delete degenerate face corrections can be made. However after this repair degenerate faces, usually with naked edges occur.

6. Zero length edge. These errors can be repaired in the same manner as repairing degenerate faces.

7. Duplicate faces. By using the “ExtractDuplicateMeshFaces” command or the Mesh Repair plug-in to delete duplicate faces we are able to leave only one of the identical faces.

Finally when we use the “Meshbooleanunion” command to combine meshes we will prevent good meshes from turning into bad meshes when we save it. In figures 20 you can see the plotted complex three dimensional model of the Golden Star System.

By keeping the center point of the arch at the desired golden mean ratio the shapes were generated after the basic Golden Star System component was built. You can see in figure 20 that an arch which can serve as a structural support itself or as a rib in a vault can be constructed by installing additional arches at geometrically compatible points much as has been done by architects over the centuries. In figure 20
the golden mean was defined to as the span of the arch. This was done to remain consistent with the initial design and to keep the proportions the same throughout the model. The type one arch (in figure 20) is constructed so that all of the topmost points of the Koch Star meet the centerline support at the same spot. It should be noted that because of the constraint of the 137.5 degree rotation per module the resulting beam or arch will be asymmetrical. A buttress like extensions can be added to the ends of the arches to accommodate this anomaly so that mounting the beam or an arch or vault configuration can be attached to the foundation. Alternatively, one may continue the rotation to a point which may be more conducive to a simple attachment to the foundation. Type two arch is constructed in a similar manner. The only difference with type one arch is that the right hand and left hand strands attach at the centerline offset by one half the distance between the precious attachments of the opposite strand on type one (figure 6 and the dark green arch in figure 1).

![Figure 20: Oblique View of Type One Arch Plotted Model](image)

**Conclusion**

Biomimicry inspired design ideas are growing faster than ever in art and architecture. Architects and artists often face challenges in creating both digital and physical models of their ideas. This project showed that new tools should be developed to generate the digital models. In addition, our project demonstrated that with the dominance of geometrically complex ideas, the need for a software that makes the model ready for 3D printing is more than ever before.

**Bibliography**

**Introduction**

Mathematical art is art inspired by mathematics. My favorite example is the Mobius band. For mathematicians, there is generally one typical Mobius band obtained by giving one end of a strip of paper a half twist and then joining the ends of the strip. The result is a one-sided surface. Actually there are two possible bands if one considers a right half twist or a left half twist. The idea of a Mobius band has inspired a large variety of Mobius band sculptures where the variations occur in shape, dimensions, and material such as wood or stone. The Japanese sculptor Keizo Ushio has carved many Mobius band sculptures in granite and his work is discussed in [2-6]. Larry Frazier is an American sculptor who has also carved a variety of Mobius band sculptures in wood and alabaster and we have discussed his work in [1]. Our purpose here is to discuss a selection of recent works by Frazier. All photos are by Frazier.

**Mobius Band Sculptures**

The first Mobius band sculpture is shown in Figure 1(a) and is carved in alabaster. The second example is shown in Figure 1(b) and is carved in wood.

![Figure 1](image)

Figure 1. (a) Tuscan alabaster, h 28 x w 8.7 x d x 5 in. (b) Masur Birch, h 29 x w 9 x d 8 in.

In Figure 1(b) we are looking down on the half twist, which is concave to the right at the upper part and concave to the left at the lower part. These concavities merge into a space under the half twist. If we look at Figure 1(a) from the right, we would have a similar view of the half twist except it would be concave to the left at the upper part and concave to the right at the lower part. The concavities also merge into a space below the half twist. This is the general structure of a typical Mobius band. Namely, a reverse concavity due to the half twist and the concavities merging into a space below the half twist.

Now consider the Mobius band carved out of a Manzanita root in Figure 2. The half twist can be seen in the top view in (a). There is a concavity at the lower left and a
concavity at the upper right, which are visible in front and back views (c) and (d). In (b) one can see the range of colors. The top in (d) is at the lower right in (b). This Mobius band combines rough surface parts with smooth surface parts, which is a nice contrast. The colorful form space sculpture also reminds me of slot canyons.

(a) (b) (c) (d)

Figure 2. Manzanita, h 11 x w 10 x d 6 in.

A very interesting Mobius is shown in Figure 3. Figure 3(a) is the view of the half twist. With the help of (b), one can trace the surface and see that it is one sided. Another view is shown in (c). This example shows how varied the shape of a Mobius band can be.

(a) (b) (c)

Figure 3. Domestic hardwood, h 11 x w 10 x d 6 in.

One can also start with a U-shaped piece of paper and connect the ends with a half-twist to obtain a one-sided surface. An example of this shape in alabaster is shown in Figure 4. A different shaped example of a one-sided surface is shown in Figure 5. The half-twist is on the right.
Two Sided Surfaces

If a strip of paper is given a full twist before joining the ends, then the resulting band will be two sided. An example is shown in Figure 6 (a). This band has a figure 8 shape as seen in the view in Figure 6 (b).

A second example of a two-sided surface is shown in Figure 7. In this case, the band has two half-twists, which also results in a two-sided surface. This sculpture is quite elegant.

If a strip of paper is given three half-twists and the ends are joined, then the result is also a one-sided surface, sometimes called a triple twist Mobius. In fact, any odd number of half-twists results
in a one-sided surface. For a triple twist Mobius, it can also be seen that the single edge forms a trefoil knot. A beautiful example of a triple twist Mobius is shown in Figure 8.

Figure 7. Mun Ebony, h 28 x w 16 x d 7.5 in. Triple Twist Mobius

Figure 8. Triple Twist Mobius, Edge is a trefoil knot, Tuscan Alabaster, h 23 x w 14 x d 4 in.

References


Movable Sculptures

Dr. Michaella Janse van Vuuren
NOMILI
P.O. Box 1799
Faerie Glen
Pretoria, 0043, South Africa
E-mail: michaella@nomili.co.za

Abstract

Digital design and additive manufacturing makes it possible to create objects that are impossible to fabricate by traditional means. Through this method I have been able to create freeform sculptures movable components fabricated with no assembly required. This paper describes the process of creating the artworks from the initial idea to the digital sculpting process and additive manufacturing. The Horse Marionette, The Rocking Springbuck and Birdman sculptures are discussed in detail.

Introduction

I am a designer and artist with a PhD in Electrical Engineering. These diverse interests enable me to create sculptures that are technically complex and artistically competent. The paper relates the design process from initial idea to final sculpture, describing the technical issues involved while designing for additive manufacturing. Three of my sculptures the Horse Marionette, the Rocking Springbuck and the Birdman will be discussed in detail.

Artists Background

I completed my schooling at Pro Arte, the School for Music Art Drama and Ballet in Pretoria, South Africa. After some years spent as a freelance and performance artist I commenced my studies in Electrical Engineering at the University of Cape Town. In 2004 I obtained a PhD in Electrical Engineering. My computer vision thesis focused on human pose and action recognition using negative space analysis [1]. This was followed by a year as researcher at the Centre for Scientific and Industrial Research (CSIR) and a Postdoctoral fellowship in medical custom implant design using CAD and additive manufacturing at the Central University of Technology in Bloemfontein. During this time I fell in love with the possibilities of 3D printing and the unique potential of this manufacturing process to merge my artistic and technical interests. I started my company, NOMILI (www.nomili.co.za) in 2008, to exclusively dedicate my time to design for 3D printing.

Related work

Non-assembly mechanisms have been an area of research for a number of years [2, 3]. A few designers have incorporated the mechanical potential of additive manufacturing into their work. Notable designs are The Bloom [4] adjustable lighting design and One-Shot [5] folding table by Patrick Jouin. The Volume [6] light by Dror is printed as a single piece and expands from a completely flattened position to produce a
fabulous display of shadow and light. Theo Jansen moved his amazing Strandbeest [7] dynamic sculptures into the world of 3D printing. All these designs are sintered in one piece and are capable of transforming into another shape. These parts require more planning than static objects. The work introduced in this paper not only shares the mechanical complications of the work referenced above but also adds an organic figurative aspect to the design.

Design Method

The flowchart in Figure 1 shows an overview of the design process from the initial idea to the manufactured piece. I create my sculptures by first visualizing the object. This is then translated onto paper, and after a series of sketches transformed into a technical drawing. This drawing dictates the measurements and dimensions needed to translate the artwork into the computer. Many of my designs include movable parts. The scale of the sculptures, gap between parts and mechanical functionality has to be planned meticulously before transfer to the computer. An additional complication is that the hinges and movable parts have to be visually pleasing and not just functional.

Different software programs are used to convert the idea into a printable digital design. These packages specialize in specific types of functionality while no one program exists that fulfills all the design needs. The digital freeform creation is done by utilizing Zbrush [8] and an Intuos4 tablet. Zbrush is a sculpting program that allows organic freeform sculptural shapes that are very difficult to create using traditional CAD software.

When sculpting the freeform shape the internal mechanism of the final design has to be kept in mind. Enough space must be allowed to ensure that the mechanism can be incorporated into the design at a later stage. I use Rhinoceros 3D[9] when there is a need to cut and perform Boolean operations, or when adding mechanisms with precise dimensions. Wings3D [10] and T-Splines [11] can also be useful.

Figure 1: Flowchart of the design method.
I alternate between the CAD design and the technical drawing phase to create the 3D model. As the work progresses it is often the case that once a digital model exists, potential challenges become clear. The 3D model can be viewed from many angles to show areas where parts may fuse or collide. It is then necessary to return to the technical drawing and to rethink the mechanism. In this manner feedback loops exists between the various stages of the design. Ultimately, if an error shows up in the final design a redesign at the technical drawing stage will be necessary.

When designing for additive manufacturing the limitations of the printing process must be taken into account. Unlike designing for computer rendering, a visual representation, modeling for 3D printing requires careful planning before it is ready for 3D printing. The object needs to be watertight and manifold with no bad edges, to name but a few. Software programs such as Netfabb[12], Meshlab[13], MiniMagics[14] and Magics[15] help to evaluate and repair 3D files, or meshes, for the file to be printable. It helps to check your file throughout the design process for printing integrity as many hours or days of work can be lost trying to fix a file that has become unprintable. One also needs to comply with the process limitations. Some additive manufacturing processes allow for moving parts, while others require that the object be designed as a single solid piece. All of the sculptures presented here were built in Polyamide using Selective Laser Sintering. This Nylon material has the texture of coral and is well suited to creating movable parts. Parts designed too small will disintegrate in the build, and movable parts cannot be spaced too close to one another since they may fuse and therefore will not be able to move at all. I use a minimum gap width of 0.7mm between working parts, and a thickness of at least 1mm for parts that are not load bearing. It is important to hollow parts not only to make space for the internal mechanisms but also to reduce printing costs. A wall thickness of 2mm is usually sufficient. Open spaces have to be incorporated into the design to allow excess powder trapped inside the hollowed shape to escape and to allow the internal mechanisms to function properly. Build orientation of the model is also important; more detail is traced horizontally by the laser than in the vertically fused layers.

The design process is time consuming and relatively expensive. As a result I have to design these sculptures to be as small as possible always pushing the limits. There is no room for error; one small mistake in the mechanics could result in the entire design not operating as intended. The design is a big challenge, it is difficult to imagine how all the parts would move and interact with one another without having a physical model to inspect.

When a design is completed, it is emailed to a local or international additive manufacturer. The sculptures are built up by fusing thin layers of nylon powder. When the print build is finished the powder is removed and the object magically emerges from the heap of deposited powder. A few days later the completed sculptures arrive at my door ready to be unpacked. This is always a very tense moment. Did I think of everything? Will the result be as planned? All the planning and designing centers on this one moment of pure joy when I hold an object that looks and functions exactly as I envisioned it.

Results

The following sections discuss in detail how the methodology was applied to produce three different sculptures, The Horse Marionette, The Rocking Springbuck and The Birdman.

The Birdman. The Birdman sculpture was inspired by automata and mechanical toys. When the rod is pulled upwards the beak closes. When the rod is pushed down his beak opens and the arms move outward. The Birdman’s body and lower beak was sculpted separately in Zbrush. The body was then hollowed with a wall thickness of 2 mm. Rhinoceros 3D was used to separate the body parts and to create the internal mechanism and the chainmail wings. Figure 2 shows the internal mechanism of the Birdman. Figure 2.2 and 2.3 show cross sections of the Birdman’s beak. When the rod is pulled a small bar mounted on the rod
pushes the Birdman’s lower beak upwards and closes the mouth, when the rod is pushed down the bar moves downwards and the beak opens again. To lift up the arms the rod is pushed downwards, this pushes on the bottom rings that in turn pull on the levers that swivel the arms upwards. Figure 3 shows the final printable file, the Birdman’s mechanism and the interlinked chainmail emerge from the 3D printer in its final form with no assembly required. Figure 4 shows photographs of the final sintered design. A video of the Birdman in action can be seen online [16].

![Figure 2: Internal mechanism of The Birdman.](image)

![Figure 3: The Birdman, render of the final printable 3D design file.](image)

**The Rocking Springbuck.** The Rocking Springbuck is dedicated to a springbuck I hand reared and also a commentary on wild animals in captivity. When the Rocking Springbuck is pushed the rocking action sets a pendulum in motion that rotates the gears mounted in the Springbuck’s body. The buck’s head is placed on a spring that allows the head to move as the buck rocks. Figure 5 shows the body of the sculpted buck in Zbrush. The buck was hollowed in Netfabb and adapted in Rhinoceros 3D. The weight of the buck had to be distributed to ensure the volume centroid is in the centre of the buck. Due to time constraints the gears were not built from scratch but an existing open source gearbox model was modified [17]. Figure 6 shows a cross-section of the printable buck. Multiple views of the printable 3D files are shown in Figure 7. All the parts of the buck have been placed in the same 3D file so no assembly is required, and the sculpture emerges from the 3D printer with all the movable parts in place. The Springbuck has to be
orientated on its’ side when sintered. This orientation ensures that the gears are laser fused with the greatest amount of detail in the gear teeth. An upright position previously resulted in warping and the gears not revolving smoothly. A video has been recorded of the buck in motion [18].

**Figure 4:** The Birdman, 22x60x40mm.

**Figure 5:** The springbuck in Zbrush.
Figure 6: Internal workings of the Rocking Springbuck.

Figure 7: The Rocking Springbuck, renders of the 3D printable file.

Figure 8: The Rocking Springbuck dimensions 177x163x53mm.
The Horse Marionette. The Horse marionette was a personal project, homage to my earlier years of puppet making. The challenge was to create something that was not only beautiful but pushed the technological boundaries and challenged my 3D skills.

Figure 9: The Horse Marionette sculpted in Zbrush.

Figure 9 show the basic horse shape sculpted in Zbrush. The sculpted horse was then transferred to Rhinoceros 3D where the body was separated into the different parts that make up the marionette’s head, neck, torso and leg joints. Figure 10 shows cross sections of the final horse design illustrating the layout of the internal mechanisms. A lot of thought went into the design of the movable parts. Enough space had to be left between parts to make allowance not only for the marionette’s movement but also to ensure that the parts do not fuse during the sintering process. Figure 10.2 shows a cross section of the head, which has a barrel hinge at top of the neck. A triangular inset was added to the bottom of the head to create a visually continuous result when the head is lifted. The bottom of the neck hinges to the top of the body, a 3 mm gap was left between these parts to ensure that the area does not fuse. Powder gaps were made at the bottom of the neck and the bottom of the body to ensure that the excess powder escapes. The tail, shown in Figure 10.3 was constructed using T-Splines. A cage structure surrounds a ball mounted at the end of the following cage. The ball is placed at the centre of each structure to ensure it does not fuse. The gaps surrounding the ball are too small for the part to fall out. When the print is finished and the powder removed the tail emerges as one loosely connected movable part as seen in Figure 12.
Figure 10: Cross-section renders of the Horse Marionette.

Figure 10.4 and 10.5 show a cross section of the thigh and trunk of the body. The upper thigh mechanism was a complicated part to construct due to the small space available for the hinge and the large rotation that the leg undergoes. To avoid a collision with the trunk and upper thigh enough space had to be left between the areas to allow for maximum movement as shown in Figure 10.4. Figure 10.5 illustrate how most of the mechanism is hidden inside the thigh. An open space was left at the bottom of the thigh to allow powder to escape. Figures 10.6 and 10.7 shows the knee and ankle joints. These joints not only have to function properly but also resemble the corresponding anatomy. The horse’s wings shown in Figures 10.1 and 10.3 were made out of interlinked shapes that resemble the feather pattern on a bird’s wing. The interlinked pattern fans out when the wings are lifted and pack together when the wings are dropped.

Figure 11: Different views of the final printable 3D design.

Figure 11 shows the front, side and top view of the final printable 3D design. All the horse’s parts have been placed in the same printable file so no assembly is required afterwards. The sintered Horse Marionette is shown in Figure 12. The first time I held the design in my hand was when the finished marionette arrived. The images show the marionette exactly as it came from the printer. The planning and
designing focuses on this moment when I hold a design that looks and functions exactly as I envisioned it. The Horse Marionette has fully functional joints and movable wings, when it is strung up the horse come to life. Figure 13 shows images of the animated horse. A video of the horse in action can be seen on the internet [19].

**Figure 12:** *The sintered Horse Marionette 100x193x203mm.*

**Figure 13:** *The Horse Marionette 100x193x203mm.*

**Conclusion**

This paper presents a design workflow that combines freeform shapes and non-assembly mechanical parts to create movable sculptures. Three sculptures, the Birdman, Rocking Springbuck and Horse marionette illustrates different applications of the workflow.

**Future work**

In the next stage I would like to combine electronics, lighting and motors to animate and illuminate the objects.

**References**


Near the end of his life the Nuremberg artist and publisher Albrecht Durer printed two books compiling his applications of geometry and analytical thought to drawing. A number of these anticipate applications in today’s computing. Among these are the invention of the morphing or transformation grid used in image processing, the devising of a method for generating facial features used in facial recognition software and the creation of a perspective drawing machine that is the analog of ray tracing. Durer also invented specialized drawing instruments, one of which is a primitive analog computer.

**Introduction**

In 1525 Albrecht Durer published his famous text on applied geometry "The Four Books on Measurement: Instructions for Measuring with Compass and Ruler". A few years later, just after he died in 1528 at the age of 56, his widow released "The Four Books on Human Proportion". Both books assemble cutting edge knowledge at the time, but also hold numerous innovations in applied mathematics. As it turns out, a number of these innovations remain essential to modern day applications of computing

“On Measurement” was written as a text for young artists laying out the geometric knowledge that Durer believed necessary for professional art making. Largely devoted to geometric construction of polygons and numerous curves, it also boasts sections on perspective. Especially intriguing in their presaging of current computing applications are Durer’s illustrations of devices he created for perspective drawing and the drawing of curves.

“On Human Proportion” largely treats human physiognomy, ostensibly the study of human character as reflected in bodily and facial features. Physiognomy’s spurious assumption of causality between physical features and character did not concern Durer. Rather his book was an early treatise on anthropometrics, a set of human measurements standardized from a study of a number of representatives from three different age groups of both sexes. Unique in this book was Durer’s adaptation of the methods of physiognomy into mathematical methods for the artist to generate new faces from a single type. These methods remain today in facial recognition programs and in image processing programs.

**Scientific Background**

Throughout his life Albrecht Durer investigated the role of descriptive geometry in accurately depicting visible reality in drawing. Like the Italians of the *disegno* school he held drawing to be the method for uncovering the conceptual and technical tools to create art of appropriate depth. He wrote: “Sane judgment abhors nothing so much as a picture perpetrated with no technical knowledge, although with plenty of care and diligence.”

His investigations included two extended trips to Italy, where his exposure to the Venetian art world of 1494-95 propelled his interest in mathematics. Acquaintance with Giovanni Bellini, for example, brought him up to date with advances in perspective. Also, during his stay there Luca Pacioli published the first printed book on mathematics, “Summa De Arithmetica, Geometria, Proportioni Et Proportionalita”.
Pacioli, a Franciscan friar and a student of Della Francesca, wrote “Summa” as an extensive compendium of mathematics as known in the late 15th century. Though he did not then meet Pacioli, Durer consequently updated his knowledge of mathematics and learned of Leonardo Da Vinci, Pacioli’s housemate, and his application of mathematics to art.

Ten years later Durer returned to Italy and on this trip did meet with Pacioli. The most important outcome of this and his earlier Italian sojourn was to introduce Durer to scientific methods. His embrace of science went on to condition his thinking for the rest of his life.

Figure 1. Albrecht Durer and Johannes Stabius, Star Map: Northern and Southern Hemispheres, 1515

One example of this was Durer’s abandonment of astrological subject matter, a big seller for a printer of the time, in favor of astronomy. This was not to be a casual interest. In 1509 he purchased the entire library of astronomer Regiomontanus from the estate of Nuremberg businessman Bernhard Walther. Walther had sponsored the residency of Regiomontanus in Nuremberg between 1471 and 1475. Regiomontanus was then the leading scholar of Ptolemaic astronomy, a noted mathematician and a designer of sophisticated new instruments for improved astronomical measurements.

In 1515 Durer became the designer and publisher of the first printed star map in Europe (Figure 1). Produced in collaboration with the Austrian cartographer and mathematician Johannes Stabius, the map relied heavily on data assembled by Regiomontanus, plus refinements developed by Walther.

Along with Durer’s knowledge of the Ptolemaic system came an understanding of a family of curves known as epicycloids. A fixed point on a circle will trace an epicycloid as that circle rolls around the perimeter of yet another circle. By varying the relative size of the two circles and the position of the fixed point relative to the rolling circle a host of unique curves are possible (Figure 2). These curves will often loop backward, as they seem to coil around a circle.

The fact that epicycloids generate from two circles was key to the Ptolemaic description of planetary motion. To the ancient mind it was a given that all celestial motion must be perfectly circular. Nevertheless the observed paths of the planets, as seen from earth, were anything but circular. To the
contrary, planets would appear to stop and then loop backwards before proceeding forward. Claudius Ptolemy, a Greco-Roman astronomer and geographer, explained this retrograde movement by describing the path of the planets as epicycles, i.e., circles moving upon circles (Figure 3). Thus the perfect motion of the circle could be retained to explain the motion of the planets in a geocentric orbit.

![Figure 2. Sample epicycloids.](image1)
![Figure 3. Planetary system of Claudius Ptolemy.](image2)

**Curves and Analog Computing**

In the first book of “On Measurement” Durer illustrates an instrument of his own design used to draw epicycloids. This instrument – a compass of sorts for drawing circles upon circles – consisted of a sequence of four telescoping arms and calibrated dials (Figure 4). An arm attached to the first dial could rotate in a full circle, while a second arm fixed to another dial mounted the end of this first arm could rotate around the end of the first arm. This rotation around a rotating point described an epicycloid.

![Figure 4. Durer's epicyclical compass.](image3)
![Figure 5. The limaçon of Pascal as drawn by Durer.](image4)
As the son of a goldsmith Durer possessed the knowledge of metalworking to shepherd his design through the shops of the world-class instrument makers of Nuremberg. Precision calibration and adjustable arms allowed its user to plot an endless number of curves by setting the length of each arm and determining the rate at which the arms turned. This, in effect, constituted a manual programming by presetting the parameters of each curve plotted.

The example given in “On Measurement” is a curve known to mathematicians as the limaçon (snail) of Pascal (Figure 5), although Durer published the curve some 100 years before Étienne Pascal. To draw this curve Durer used two arms of his epicyclical compass. Each time he rotated the central arm $ab$ in 30° increments, he simultaneously rotated the second arm $bc$ in 30° increments in the same direction. Interpolating the plotted points into a smooth curve completed the construction. The curve approximates one rotation of the planets Mars and Venus, and may have been the artist’s rationale for constructing it.

Durer’s instrument could also draft a family of closely related curves known as the hypocycloids. These curves generate from a point on a circle rolled inside of a stationary circle. These are the curves created with the classic drawing toy, the Spirograph®.

Durer’s Door and Raytracing

Among Durer’s mathematical capabilities was an understanding of projective geometry unparalleled by any artist of his time. His involvement in the star map of 1515 and an earlier publication of the 8th century Persian astronomer Mashallah ibn Athari’s treatise on the astrolabe introduced him to stereographic projection. His second trip to Italy included meetings with Pacioli familiarizing him with Piero Della Francesca’s unpublished work on perspective. At the time of his death he was working on a book dedicated to this geometry.

Figure 6. Proportioning text for perspective correction. Figure 7. Durer’s portable “scanner”.

Some of this knowledge appears in books three and four of “On Measurement”. Book three focuses on monument design and contains sections on the anamorphic correction of proportions in columns and lettering (Figure 6) as well as on solar projection in sundials. Book four features some of the most widely reproduced illustrations from “On Measurement”. These depict a number of apparatuses devised and sold
by Durer as aids for perspective drawing. One, depicted in Figure 7, he referred to as a scanner. This was a portable apparatus meant for use in the field or in a portrait subject’s home. Variations on this device were available commercially until the 1930’s.

![Figure 8. Durer’s door.](image)

Arguably the most intriguing of Durer’s apparatuses was a device employing the same principles as raytracing, the most common method of rendering 3D images in a computer. Raytracing measures the raster position of a ray emanating from a point on a virtual object to the position of a virtual camera as that ray passes through a virtual screen. The sum total of these raster points, i.e., pixels, creates a two-dimensional mapping of the object.

Durer’s door, as the device came to be named, constitutes an ideal physical analog of raytracing. Figure 8 demonstrates how the device works. A thread attaches to the end of a stylus that an assistant moves from point to point along the contours of the object, in this case a lute. The thread passes through the door’s frame and then through a hook on the wall. A weight at the end of the thread ensures that it remains taut as the stylus changes position. The thread forms a ray and the hook acts as the center of projection and corresponds to the camera position in raytracing.

Figure 8 has Durer using a square to measure the horizontal and vertical position at which the thread passes through the door. He then would transfer this measure to a paper sheet affixed to the door. The image that emerged as dotted lines displays near perfect perspective

**Physiognomy and Facial Recognition**

Physiognomy was a now debunked science, which purported to reveal connections between one’s facial characteristics and one’s psychological characteristics. Its reach dated back to a treatise by Aristotle and
continued until fizzling out in the 19th century. Its methods aped science in that practitioners carefully measured human faces and categorized these according to the proportions relating the measures.

Artists were expected to know something of physiognomy. Its use in late Medieval and early Renaissance art was cautionary: used in illustrating religious broadsides to represent all manner of sinners and their evils, which the faithful were to avoid. It was also precautionary: artists must be aware of physiognomy in order to avoid potentially negative depictions of saints or of one’s patrons.

In “On Human Proportion” Durer borrowed from the methods of physiognomic measure as developed by Leon Battista Alberti and published in Francesco di Giorgio's “De Harmonica Mundi Totius” of 1525. He determined “average” faces by measuring a sampling of over 200 people from five groups as determined by age and gender. These included old and young men and women and infants. He then justified his anthropometric findings with Vitruvius’ canon of proportions to create an ideal standard for each group. In this regard Durer wrote: “I hold that the perfection of form and beauty is contained in the sum of all men.”

Durer then drew grid lines over orthographic views of the head such that intersections denoted over twenty significant features of the face. By moving the grid points and thus the position and proportions of the original, the artist could generate a new version of the face (Figures 9 and 10).

The number of possible versions makes for some interesting numbers, which become astonishingly large. If each of twenty points were to move only a single unit in horizontal, vertical and diagonal directions, then the possibilities are $9^{20}$ or 12,157,665,459,056,928,801. If the changes in position increase to two units in each direction, then the variations increase to $25^{20}$ – a number too tedious to type. It becomes clear why today’s facial recognition programs use Durer’s method. A typical facial recognition program processes data from 25 points on the face (Figures 11, 12 and 13).

**Morphing Grids**

In 1524, prior to the publication of di Giorgio’s “De Harmonica”, Durer invented another method for transforming faces. (The evidence for this is found not found in “On Human Proportion”, but in Durer’s sketchbooks.) Known today as a transformation grid or morphing grid, this method appears as a common tool in image processing programs.
Durer drew a head overlaid by a standard raster grid. Rather than shifting the position of key grid lines, he subsequently distorted the grid by elongating and flattening it as well as skewing and tapering the grid in four directions. Figure 14 reconstructs Durer’s transformation grids by using the Envelope tool in the draw program Inkscape. Figure 15 illustrates a distortion of Durer’s self-portrait using a mesh – a grid with freely moving points – from Corel’s Photo-Paint program.
Conclusion

In the very last years of his life Durer wrote on his ideas of aesthetics. Most notably he believed that any notion of beauty was at best elusive, in large part due to the complexity of any artistic endeavor. He decreed, however, that the artist should “insist on the genuine forms of nature”. Key among those forms was geometric order, which Durer, like the Italians Pacioli, DaVinci and Della Francesca, believed to be the reflection of the mind of God in nature. Part and parcel of nature was the ordering of human vision, where geometry also governed.

To Renaissance mind the act of drawing, or disegno, was the primary artistic and conceptual means by which nature’s embedded geometry was initially revealed and recorded. Durer sought to expand the ideas of descriptive geometry and its application in descriptive drawing by adding to the conceptual tools these required. Any art that had not such tools, i.e., those that elicit genuine forms, he found abhorrent.

In his search for these newer and better tools Durer devised methods that continue today in visual computing. The ability to produce imagery by application of descriptive tools, tools of hand and mind, to drawing remains invaluable to the production of digital art.
Introduction

My work was previously discussed in [1]. Here I will describe three relatively recent projects: Minimal Surface, Sun Dial, and Geodesic Sphere. Additional information also appears in the youtube videos [2-4].

Minimal Surface

Minimal Surface is a new sculpture of mine that is located at School for Children with Special Needs in Serres.

Figure 1. Minimal Surface, 2012, 4 x 4 x 1.5 meters, School for Children with Special Needs, “Perioxh Aidonia”, Serres, Macedonia East, Greece.
The design of this sculpture was influenced by the work of Charles Perry and Carlo Sequin. We begin with a double loop, as shown in Figure 1. A network of thin struts is added to indicate the corresponding minimal surface that has the double loop as boundary.

Additional views of Minimal Surface are shown in Figure 2. In order to understand the construction, we start with a ring indicated in red in Figure 3(a). We divide it into three equal parts and attach three metal crosses perpendicular to the ring, as in Figure 3(b). We join the consecutive edges of the crosses with circular arches of different diameters, as in Figure 3(c). There are three “windows” in the sculpture, as shown in Figure 3(d).

For building them, I used 3D elastic ellipses made of cardboard (Figure 3(e)). The dotted line of the ellipse runs along the basic ring, while the ellipse turns 90 degrees from edge to edge. The ellipses meet each other perpendicularly at each cross. The metal net was then built using a lot of measuring. The basic ring was then removed. There are also corresponding Youtube presentations [2,3].
Sundial

The sundial, another new sculpture of mine, is located at a natural attraction Alistrati’s Cave, as shown in Figures 4 (a) and (b). Front and back views are shown in Figures 5 (a) and (b). The construction of the sundial in Figures 4 and 5 is described in detail in the Youtube presentation [4].

Figure 4. Sundial, 2013, h 1.7 x w 1.3 x d 0.6 meters, Alistrati’s Cave, Serres, Macedonia East, Greece.

Figure 5. (a) Sundial construction.  
Figure 5. (b) Back view.
Irregular Spherical Geodesics

Geodesic Spheres are a new series sculptures of mine. Typically a geodesic dome design begins with an icosahedron (20 triangles) or a dodecahedron (12 pentagons) inscribed in a hypothetical sphere. But what happens if we inscribe a totally irregular polygon into a sphere. The basic rules we have to apply are as follows.

Rule 1. (Cutting arcs) The inner and outer actis (radius) are the same for all arcs (Figure 6).

The angle can vary from arc to arc. (Figure 7). Smaller angles require more arcs and have thicker spherical nets.

Rule 2. (Joining arcs) All the arcs must be on great circles (Figure 7). Thus the joining lines of each 2 geodesic arcs must be pointing to the center of the sphere. (Figure 8).

A detailed example is shown in Figure 9. The arcs are not glued together but rather hinged. The geodesic sphere is rigid due to partial triangulation, as seen in Figure 9. Two views of a geodesic sphere is shown in Figure 10. The title is Celine (moon in Greek). The sphere is not totally “closed”, so Celine is not a full moon!

References


*Figure 11. Celine, 1.2 meters, diameter.*
**Books of Interest**

*The New Mathematics of Architecture*
by Jane and Mark Burry,
Thames and Hudson, 2010.

Contents


This book has 628 illustrations with 435 in color. The authors discuss topics of contemporary mathematics and how they relate to the design and construction of cutting edge buildings. For example, the Australian Wildlife Health Center in Healesville Sanctuary, Australia; the Sagrada Familia in Barcelona, Spain; the Disney Concert Hall in Los Angeles, California; the Qatar Education City Convention Center in Doha, Qatar; the Water Cube in Beijing, China, as well as many more amazing examples. This is a very extensive presentation of striking applications of mathematics in architecture.

*Architectural Geometry*
by Hellmut Pottmann, Andreas Asperl, Michael Hofeg, and Axel Kilian,

Contents


This textbook is a very extensive discussion of geometry as applied to the architectural design process starting from basics. The book closes the gap between technical possibilities such as CAD and an effective working knowledge of the new methods of geometric design. The topics covered are fundamental for a contemporary architectural education. The presentations are clear and complete.
An early interest in the **TRUCHET** algorithm inspired several directions for art. I was attracted by the transformation that took place when one introduced the random number generator in the program. It reminded me of the patterns that occurred on the water's surface while fishing in Puget Sound (figure 1). Time between bites was spent observing the surface patterns of water moving in and out of focus.

Water and Truchet tiles developed a natural affinity providing structure and texture for **JUMPING KOI AT LAKE TRUCHET** (figure 2). Airbrush combined with screen printing gave a tactile dimension to water. While Koi are sluggish fish they suggest movement for this mythical lake. In art one can deal with fantasy as well as reality and often need a thoughtful balance.

Artists differ in their development of a visual vocabulary. They may pursue a linear direction or focus on a single reference. My own art is multidimensional and perhaps best compared to a tapestry with threads from many directions drawing on a broad base in education and experience.

An invitation to participate in a national book competition exhibition led to **LABYRINTHS OF THE MIND**. (see Figure3). I had been interested in current brain research and expected to pursue that direction. Gradually my focus shifted to my own experience. It introduced varied aspects of my art and thought including color research, computer experimentation, and Truchet development. travel and mentors.

The title was also selected for a major solo show at THE LEONARDO museum. Water and Truchet tiles became important in developing the art for the Nursing College. Working on **WATERWALL**
suggested a companion work **FIREWALL** and gradually the entire **ELEMENTS** series (see Figure 4) developed.

![Figure 3.](image1)

![Figure 4.](image2)

In the Greek concept, water, fire, wind and earth represent fundamental elements of life. For the entrance of Nursing College, I used four large prints of the **ELEMENTS** series, as you walk through the entrance corridor. In moving toward the major mural **LET THEIR SPIRITS SOAR** (see Figure 5), continuing the Truchet direction seemed desirable, but practically not feasible. For a 36’ mural, 3’ wide panels were used.

![Figure 5.](image3)

A new program was needed to create spacial depth. Cost prevented having 12 screens for the work, and one computer image was programmed to rotate and merge with adjacent units. The mural was essentially layers of activity with hand-painted background and natural references. The landscape was drawn from my photos of Great Salt Lake. Dancers were chosen to represent nurses for lack of unique costumes to identify them. For this work the background was painted on canvas panels, the program generated on a computer and some images transferred to a screen for printing.

**MAN AND WOMAN** (see Figure 6) paralleled Truchet tiles for this project. They occurred in a number of earlier prints and paintings in different scales and contexts. They can also be thought of as part of my visual vocabulary.

For the **LEONARDO** exhibit, some current experiments with **BODY PARTS** (see Figure 7) drawn from MRI films that were taken after I had several falls. The films were interesting, and I thought it could have limited use. To pursue the direction of the museum at that time, I developed
RENAISSANCE MEN. (see Figure 8) showing Renaissance experiments on human form. From earlier research and rare books collection of the university's Marriott Library, Vesalius figures of medical experiments were included.

Figure 7.

I think of the water study as Mythical LAKE TRUCHET (see Figure 9). It was inspired by the tile pattern created by the 18th-century French mathematician and memories of fishing on Puget Sound. For this work the background was painted on aluminum plates, the program generated on a computer and the image transferred to a screen for printing.

Figure 8.

For the benefit of viewers, I showed the actual development of the tile pattern from two simple tiles with arcs in opposite corners. The random number generator in the program produced the pattern which to me suggests the movement of water.

Figure 9.

For future work, the arts discussed indicate a range of interest. The basic technique of hand-painted grounds and technological experiments will probably continue with nature and natural forms, possibly using historical references.

References:
Bliss, Anna Campbell, "Art for a House of Mathematics" 2006, Bliss Studio Publication
Consultant for programming: Wayne Rossberg
Introduction

Upper and Lower Antelope Canyons are two of the best known slot canyons that you can walk through. They have been carved out of sandstone by the action of wind and water. Their locations are on Navajo land just outside Page, Arizona near the Utah/Arizona border. Three previous articles on slot canyons with photographs by Robert Fathauer appeared in [1-3].

Figure 1. Nat Friedman at the entrance to Upper Antelope Canyon.

On March 5, 2014, the authors hiked through the two canyons and a selection of photographs by Fathauer are presented below. The slot canyons are a photographer's dream and there is no end of possible great images. For Friedman, this was his first visit to the slot canyons. Having carved stone for over 40 years, this was a long anticipated pilgrimage to what can only be described as sculpture heaven. The slot canyons are truly amazing compositions of form, space, light, and color.
Upper Antelope Canyon

We will first consider some images from Upper Antelope Canyon. In Figure 1, one can see the relative size of the canyon compared to human size. We took a 2 ½ hour photography tour with a Navajo guide. A tripod and SLR camera were required and the cost was $80 per person. It was well worth it as the guide pointed out the best angles and positions for photos.

In Figure 2 one can see the range of forms carved out by water rushing through the canyon. The layers of stone suspended in space, one in front of another, result in beautiful images.

The camera angle in Figure 2 is raised to obtain the image in Figure 3. The stone at the top in Figure 2 appears just above center in Figure 3. Thus by a slight change in viewing direction, one obtains a very different image. This is true in general in viewing the slot canyons.

In Figure 4, one can see the flow lines etched into the stone as the water rushed through the canyon. Of course, this was a very slow process taking millennia.
A particularly interesting form is shown in Figure 5. It is hard to imagine how this form managed to survive the torrents of rushing water.

**Lower Antelope Canyon**

An interesting "wave form" is shown in Figure 6. Note the undulations of the surface on the right of the wave crest. It is like the surface of flowing water carved in stone. The range of light and color is quite delicate.

The image in Figure 7 is somewhat reminiscent of drapery on a human figure. Certain edges appear to result from the stone breaking off and it is interesting to see how the curved lines follow the shape of the stone.

In Figure 8, one can see a hole in the lower part of the stone that is referred to as Keyhole Rock. The range from light to dark is prominent in this image. This can also be said for the striking image in Figure 9.
The last image in Figure 10 is looking up at the top of the canyon with clouds and sky visible. The detail in the thin layers of irregularly shaped stone is eye-catching.

In conclusion, all I can say is that I can’t wait to go back. You really should see it for yourself.

References

